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Variable Structure Adaptive Control of Wing-Rock Motion of Slender Delta Wings

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Based on the variable structure model reference adaptive control theory, a new control system for the control of wing-rock motion of slender delta wings, using only roll angle measurement, is designed. For the derivation of the control law, it is assumed that the aerodynamic parameters and the structure of the aerodynamic nonlinear functions in the model are unknown. Moreover, it is assumed that disturbance input due to wind gust is present in the system. It is shown that, in the closed-loop system designed using bounds on uncertain functions, the roll angle tracks given reference trajectory, and the wing-rock motion is suppressed. Digital simulation results show that the closed-loop system has good transient behavior and robustness to the uncertainties and disturbance input.

Nomenclature

A, A_0, A_c, $\boldsymbol{b}$, $\boldsymbol{b}_0$, $\boldsymbol{b}_c$, $\bar{\boldsymbol{b}}_0$, $\boldsymbol{h}$, $\boldsymbol{h}_c$	= system matrices
a_i, b_i, c_i, μ_i, p	= aerodynamic coefficients
d, d_w	= disturbance inputs
$\boldsymbol{e}$, e_0, e'_i	= signal errors
f_i	= modulation functions
$g(x, t)$, $g_c(x)$	= nonlinear functions
$\bar{g}$, $\bar{g}_c$	= bounds on functions
k_m, α_{mi}	= model reference parameters
k_p	= input influence gain
$M(s)$, $\bar{M}(s)$, $W(s)$, $L(s)$	= transfer functions
r	= reference input
u, $(u_0)_{\text{eq}}$	= control and equivalent control inputs
v_i, χ_i, ξ_i	= filtered signals
$\boldsymbol{x}$, $\boldsymbol{X}$, $\boldsymbol{X}_m$	= state vectors
α	= angle of attack
Δ	= saturation function parameter
δ, γ, λ	= filter parameters
κ_{nom}, $\boldsymbol{\theta}_{\text{nom}}$, u_{nom}	= nominal parameters and input
κ^*, $\boldsymbol{\theta}^*$, u^*	= parameters and input for model matching
$\bar{\kappa}$, ρ, $\bar{\theta}_{ij}$, $\bar{g}_i$, ϵ_i	= parameters used in Table 2
τ	= averaging filter parameter
ϕ, ϕ_m	= roll and reference roll angles
ω	= measured signal

I. Introduction

THE phenomenon of wing rock is manifested by a limit cycle oscillation predominantly in roll about the body axis. Recently, several theoretical and experimental studies have been performed to understand the dynamics of wing rock and to predict the amplitude and frequency of oscillation of the limit cycle.[1–4] Approximate nonlinear aerodynamic mathematical models of swept slender wings for one and three degrees of freedom for calculating wing-rock characteristics have been developed in Ref. 2. Analytical models of the wing-rock phenomenon have been used to predict roll divergence and periods and amplitudes of oscillations in Refs. 2 and 4.

Recently, control systems for the control of wing-rock motion have been designed.[5–9] Luo and Lan[5] considered the question of control of wing-rock motion using the optimal control and the least-square method. Based on the solution of the Hamilton–Jacobi–Bellman equation, an optimal controller has been designed in Ref. 6. Adaptive and neural control systems have been developed for the wing-rock model with unknown parameters.[7,8] The backstepping design technique has been used in Ref. 8 to obtain a feedback linearizing adaptive controller for the wing-rock model including a first-order actuator dynamics. A discrete-time sliding mode controller is developed in Ref. 9 using a Taylor series expansion of the wing-rock model. Adaptive control systems developed in Refs. 7 and 8 assume that the wing-rock model does not have disturbance input and that the structure of nonlinear functions (except for the neural control[7]) is known. In the presence of external disturbance inputs and unmodeled nonlinearities, adaptation laws of integral type used in Refs. 7 and 8 can cause divergence of controller parameters. Moreover, the control algorithms developed in Refs. 5–9 use full-state (roll angle and roll rate) feedback. Thus, there is a need to design a control system that is effective in the presence of disturbance input and unstructured nonlinear functions in the model and requires only output feedback for the wing-rock control.

The contribution of this paper lies in the design of a new control system for the control of wing-rock motion of simple slender delta wings based on the theory of variable structure model reference adaptive control (VS-MRAC),[10–12] using only input and output signals. No attempt is made to control wing rock of complete aircraft configurations. Unlike the published works in the literature,[5–8] the wing-rock model can include disturbance input and unstructured nonlinear aerodynamic functions, and only roll angle measurement is required for the controller synthesis. However, it is assumed that an upper bound on the uncertain functions is given. It is shown that, in the closed-loop system, roll angle tracks given reference trajectory, and wing-rock suppression is accomplished. Interestingly, the controller is obtainable from the parameter adaptive model reference adaptive control (PA-MRAC) scheme[13] by replacing the standard integral adaptation laws with appropriate variable structure (VS) laws. However, instead of a single auxiliary error, namely, Monopoli's augmented error,[13] a set of n^* (where n^* is the relative degree of the plant) auxiliary errors are necessary.[10] Significant advantages of VS-MRAC over PA-MRAC designs are nice transient behavior, disturbance rejection capability, and insensitivity to plant nonlinearities or parameter variations. The simulation results presented show good transient response and robustness to uncertainty and disturbance input.

This paper is organized as follows. Section II presents the wing-rock model. A variable structure model reference adaptive controller is derived in Sec. III, and simulation results are presented in Sec. IV.

II. Wing-Rock Dynamics and Control

Analytical models of wing rock for slender delta wings have been developed in Refs. 2 and 4. A series of wind-tunnel experiments have been performed to understand the phenomenon of wing rock.[1]

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Table 1 Coefficients for wing-rock motion

α	c_1	c_2	a_1	a_2	a_3	a_4	a_5
21.5	0.354	0.001	−0.04207	0.01456	0.04714	−0.18583	0.24234
22.5	0.354	0.001	−0.04681	0.01966	0.05671	−0.22691	0.59065
25	0.354	0.001	−0.05686	0.03254	0.07334	−0.3597	1.4681

Subsequently, Konstadinopoulos et al.[3] developed a numerical simulation of this experiment. Nayfeh et al.[4] developed an analytical model of the roll moment that gives virtually perfect agreement with the numerical simulation. The wing-rock equation of motion for slender delta wings is given by [4,6]

$$\ddot{\phi} + p^2\phi = \mu_1\dot{\phi} + b_1\dot{\phi}^3 + \mu_2\phi^2\dot{\phi} + b_2\phi\dot{\phi}^2 + k_p u + d(\phi, \dot{\phi}, t) \quad (1)$$

where ϕ is the roll angle, d includes disturbance input and unmodeled functions, and u is the control input produced by the aileron. The aerodynamic coefficients in this equation are given by the following relations:

$$p^2 = -c_1 a_1, \qquad \mu_1 = c_1 a_2 - c_2, \qquad b_1 = c_1 a_3$$

$$\mu_2 = c_1 a_4, \qquad b_2 = c_1 a_5$$

and the values of the coefficients c_i and a_i for different angles of attack α from Refs. 4 and 6 are shown in Table 1.

Defining the state vector $\boldsymbol{x} = (x_1, x_2)^T = (\phi, \dot{\phi})^T \in \boldsymbol{R}^2$, the system (1) can be written in a state variable form as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -p^2 & \mu_1 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ k_p \end{bmatrix}[u + g(\boldsymbol{x}, t)], \qquad \phi = [1 \quad 0]\boldsymbol{x} \quad (2)$$

or

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{b}[u + g(\boldsymbol{x}, t)], \qquad \phi = \boldsymbol{h}^T\boldsymbol{x} \quad (3)$$

where T denotes transposition, the roll angle ϕ is chosen as the output variable, and

$$g(\boldsymbol{x}, t) = \frac{b_1 x_1^3 + \mu_2 x_1^2 x_2 + b_2 x_1 x_2^2 + d(\boldsymbol{x}, t)}{k_p} \quad (4)$$

We shall be interested in the trajectories of system (1) in a region Ω of the state space in $\boldsymbol{R}^2$ surrounding the origin. It is assumed that the aerodynamic parameters and the nonlinear function $g(\boldsymbol{x}, t)$ in Eq. (3) are unknown but the function $g(\boldsymbol{x}, t)$ satisfies $\sup_{t\geq 0} g(\boldsymbol{x}, t) \leq \bar{g}$ for $(\boldsymbol{x}, t) \in \Omega \times [0, \infty)$. Although the approach is applicable to wing-rock models with actuator dynamics, for simplicity it is assumed that the actuator is sufficiently fast and one can neglect the actuator transfer function.

Consider the input–output representation of system (1), where s is either the Laplace variable or the differential operator, given by

$$\phi = W(s)[u + g(\boldsymbol{x}, t)], \qquad W(s) = \frac{k_p}{s^2 - \mu_1 s + p^2} \quad (5)$$

and a reference model having input r and output ϕ_m characterized by the transfer function $M(s)$:

$$\phi_m = M(s)r, \qquad M(s) = \frac{k_m}{s^2 + \alpha_{m1}s + \alpha_{m2}} \quad (6)$$

where the poles of $M(s)$ are assumed to be stable.

The purpose is to find an output feedback control law $u(t)$ such that the roll angle tracking error

$$e_0 = \phi - \phi_m \quad (7)$$

tends to zero asymptotically when the motion of the system (2) evolves in Ω. Thus, by a suitable choice of the reference model, desirable regulation of the wing-rock motion is obtained.

III. Wing-Rock VS-MRAC System

For the derivation of the VS-MRAC law, the wing-rock model and the reference model must satisfy certain assumptions according to Refs. 10–12. These are 1) the system (5) is completely controllable and observable with known denominator degree (equal to 2); 2) $\mathrm{sgn}(k_p) = \mathrm{sgn}(k_m)$; 3) $W(s)$ is minimum phase (in this case, no zeros); and 4) the reference model has the same relative degree $n^*(= 2)$ as the wing-rock model.

For the wing-rock model, these conditions are satisfied, and $n^* = 2$. Now the following filters, which are useful for controller synthesis, are introduced:

$$\dot{v}_1 = -\lambda v_1 + \gamma u, \qquad \dot{v}_2 = -\lambda v_2 + \gamma\phi \quad (8)$$

where $\lambda > 0$, $\gamma > 0$, and v_1 and $v_2 \in \boldsymbol{R}$ are the set of real numbers. Define a vector of measurable signals as $\boldsymbol{\omega} = [v_1, \phi, v_2, r]^T \in \boldsymbol{R}^4$. The control law u is to be synthesized using only the signal $\boldsymbol{\omega}$.

It is well known that, for $g(\boldsymbol{x}, t) \equiv 0$, under the preceding assumptions 1–4, there exists a unique constant vector $\boldsymbol{\theta}^* = [\theta_1^*, \ldots, \theta_4^*]^T \in \boldsymbol{R}^4$ such that the transfer function of the closed-loop system with $u = \boldsymbol{\theta}^{*T}\boldsymbol{\omega}$ matches $M(s)$ exactly, i.e., $\phi = W(s)u = W(s)\boldsymbol{\theta}^{*T}\boldsymbol{\omega} = M(s)r$ (Ref. 13). Using Eqs. (5), (6), and (8), it is easily verified that for model matching $\boldsymbol{\theta}^*$ must satisfy $\theta_4^* = k_m/k_p$ and

$$\left[(s + \lambda) - \theta_1^*\gamma\right](s^2 - \mu s + p^2) - k_p\left[\theta_2^*(s + \lambda) + \gamma\theta_3^*\right]$$

$$= (s + \lambda)\left(s^2 + \alpha_{m1}s + \alpha_{m2}\right)$$

In the following we use the notation

$$u^* = \theta^{*T}\boldsymbol{\omega}, \qquad \kappa^* = 1\big/\theta_4^* = k_p/k_m, \qquad \tilde{u} = u - u^*$$

Note that, from assumption 2, both κ^* and θ_4^* are positive.

Defining the vector $\boldsymbol{X}^T = [\boldsymbol{x}^T, v_1, v_2]^T \in \boldsymbol{R}^4$, the system (2) and the filters (8) can be written in a compact form as

$$\dot{\boldsymbol{X}} = A_0\boldsymbol{X} + \boldsymbol{b}_0 u + \bar{\boldsymbol{b}}_0 g(\boldsymbol{x}, t), \qquad \phi = \boldsymbol{h}_c^T\boldsymbol{X} \quad (9)$$

where

$$A_0 = \begin{bmatrix} A & 0 & 0 \\ 0 & -\lambda & 0 \\ \gamma\boldsymbol{h}^T & 0 & -\lambda \end{bmatrix}, \qquad \boldsymbol{b}_0 = \begin{bmatrix} \boldsymbol{b} \\ \gamma \\ 0 \end{bmatrix}$$

$$\bar{\boldsymbol{b}}_0 = \begin{bmatrix} \boldsymbol{b} \\ 0 \\ 0 \end{bmatrix}, \qquad \boldsymbol{h}_c^T = \left[\boldsymbol{h}^T, 0, 0\right]$$

Here 0 denotes null matrices of appropriate dimensions. Now, adding and subtracting $b_0 u^*$ on the right-hand side of Eq. (9) and using the relation

$$\begin{bmatrix} v_1 \\ \phi \\ v_2 \end{bmatrix} = N\boldsymbol{X}, \qquad N = \begin{bmatrix} 0 & 1 & 0 \\ \boldsymbol{h}^T & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

one obtains

$$\dot{\boldsymbol{X}} = A_c\boldsymbol{X} + \boldsymbol{b}_c\kappa^*\tilde{u} + \boldsymbol{b}_c r + \bar{\boldsymbol{b}}_0 g(\boldsymbol{x}, t), \qquad \phi = \boldsymbol{h}_c^T\boldsymbol{X} \quad (11)$$

where $A_c = A_0 + \boldsymbol{b}_0[\theta_1^*, \theta_2^*, \theta_3^*]N$ and $\boldsymbol{b}_c = \theta_4^*\boldsymbol{b}_0$. For $u = u^*$, i.e., $\tilde{u} = 0$, and $g(\boldsymbol{x}, t) \equiv 0$, one has $M(s) = \boldsymbol{h}_c^T(sI - A_c)^{-1}\boldsymbol{b}_c$. Thus, $(A_c, \boldsymbol{b}_c, \boldsymbol{h}_c^T)$ is a nonminimal realization of the transfer function $M(s)$. Then, the system (9) in input–output form is given by

$$\phi = M(s)r + \kappa^* M(s)\tilde{u} + g_c(\boldsymbol{x}) + \varepsilon \quad (12)$$

where $g_c(\boldsymbol{x}, t) = \bar{M}(s)g(\boldsymbol{x}, t)$, $\bar{M}(s) = \boldsymbol{h}_c^T(sI - A_c)^{-1}\bar{\boldsymbol{b}}_0$, and ε is an exponentially decaying term due to the initial conditions that will be omitted hereafter whenever unessential for the theoretical development. Notice that $|g_c(\boldsymbol{x}, t)| \leq \bar{g}_c$ because $|g(\boldsymbol{x}, t)| \leq \bar{g}$ for $(\boldsymbol{x}, t) \in \Omega\text{x}[0, \infty]$ and $\bar{M}(s)$ is a stable operator.

Representing the reference model (6) as

$$\dot{\boldsymbol{X}}_m = A_c\boldsymbol{X}_m + \boldsymbol{b}_c r, \qquad \phi_m = \boldsymbol{h}_c^T\boldsymbol{X}_m \tag{13}$$

where $\boldsymbol{X}_m \in \boldsymbol{R}^4$ and defining the state vector error $\boldsymbol{e} = \boldsymbol{X} - \boldsymbol{X}_m$, one has the following error equation:

$$\dot{\boldsymbol{e}} = A_c\boldsymbol{e} + \boldsymbol{b}_c\kappa^*\tilde{u} + \bar{\boldsymbol{b}}_0 g(\boldsymbol{x}, t), \qquad e_0 = \boldsymbol{h}_c^T\boldsymbol{e} \tag{14}$$

Using Eq. (14), the tracking error can be written as

$$e_0 = \kappa^* M(s)\tilde{u} + g_c(\boldsymbol{x}) \tag{15}$$

For linear systems of arbitrary relative degree n^*, the VS-MRAC system design problem has been solved by Hsu[10] and Hsu et al.[11,12] In contrast to the case $n^* = 1$, the structure of the VS-MRAC is not so easily obtainable from the PA-MRAC.[13] In fact, it is necessary to introduce a chain of auxiliary errors (e_i') and a chain of modulated relay loops (Fig. 1).

One well-known restriction in the case $n^* > 1$ is that $M(s)$ cannot be chosen strictly positive real (SPR). To overcome this difficulty, a polynomial $L(s)$ of degree $N = n^* - 1$ is chosen such that $M(s)L(s)$ is SPR as in the case of PA-MRAC.[13] For the wing-rock problem, following Ref. 12, we define a polynomial $L(s)$ of the form

$$L(s) = s + \delta, \qquad \delta > 0 \tag{16}$$

Now we introduce the following sets of filtered (vector) signals:

$$\chi_0 = L^{-1}u, \qquad \boldsymbol{\xi}_0 = L^{-1}\boldsymbol{\omega} \tag{17}$$

where $\boldsymbol{\xi}_i = (\xi_{i1}, \ldots, \xi_{i4})^T \in \boldsymbol{R}^4$ $(i = 0, 1)$ and by convention $\chi_1 = u$ and $\boldsymbol{\xi}_1 = \boldsymbol{\omega}$. These signals are used to generate a chain of auxiliary errors e_i' $(i = 0, 1)$, each one governed by an SPR transfer function, namely, $M(s)L(s)$ for e_0' and $L^{-1}(s)$ for e_1', and a chain of modulation functions f_i, $i = 0, 1$. The complete algorithm for the wing-rock control is given in Table 2.

In Table 2, $\boldsymbol{\theta}_{\text{nom}}$ and κ_{nom} are nominal values of the parameters $\boldsymbol{\theta}^*$ and κ^*, respectively, obtained from some nominal model of the plant, and the upper bounds $\bar{\theta}_{ij}$ $(i = 0, 1$ and $j = 1, \ldots, 2n)$, $\bar{\kappa}$, and $\bar{g}_i$ $(i = 0, 1)$ for $\boldsymbol{x} \in \Omega$ are defined as

$$\bar{\theta}_{0j} > \rho\left|\theta_j^* - \theta_{j,\text{nom}}\right|, \qquad \bar{\theta}_{1j} > \left|\theta_j^* - \theta_{j,\text{nom}}\right|, \qquad \bar{\kappa} > |\rho - 1|$$

$$\bar{g}_0 > \sup_{t\geq 0}(k_{\text{nom}}ML)^{-1}g_c(\boldsymbol{x}, t) = \sup_{t\geq 0}(k_{\text{nom}}ML)^{-1}\bar{M}g(\boldsymbol{x}, t) \tag{18}$$

$$\bar{g}_1 > \sup_{t\geq 0}\theta_4^* M^{-1}g_c(\boldsymbol{x}, t) = \sup_{t\geq 0}\theta_4^* M^{-1}\bar{M}g(\boldsymbol{x}, t)$$

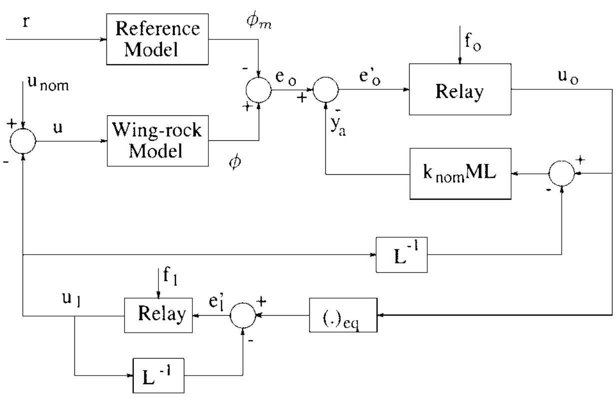


Fig. 1 Closed-loop wing-rock VS-MRAC system.

Table 2 VS-MRAC algorithm

Auxiliary errors

$$y_a = \kappa_{\text{nom}}ML\left[u_0 - L^{-1}u_1\right]$$

$$e_0 = \phi - \phi_m, \qquad e_0' = e_0 - y_a, \qquad e_1' = (u_0)_{\text{eq}} - L^{-1}(u_1)$$

Modulation functions

$$f_0 \geq \bar{\kappa}\left|\chi_0 - \boldsymbol{\theta}_{\text{nom}}^T\boldsymbol{\xi}_0\right| + \sum_{j=1}^{2n}\bar{\theta}_{0j}|\xi_{0j}| + \bar{g}_0 + \epsilon_0$$

$$f_1 \geq \sum_{j=1}^{2n}\bar{\theta}_{1j}|\xi_{1j}| + \bar{g}_1 + \epsilon_1$$

Control laws

$$u_i = f_i\,\text{sgn}(e_i'), \qquad i = 0, 1$$

$$u = -u_1 + u_{\text{nom}}, \qquad u_{\text{nom}} = \boldsymbol{\theta}_{\text{nom}}^T\boldsymbol{\omega}$$

where $\rho = \kappa^*/\kappa_{\text{nom}}$. Note that $(k_{\text{nom}}ML)^{-1}\bar{M}$ is a strictly proper stable transfer function, $\theta_4^* M^{-1}\bar{M}$ is a proper stable transfer function, and $g(\boldsymbol{x}, t)$ is a bounded signal. Note that y_a can be interpreted as a predicted output error and, hence, e_0' is a prediction error.[11]

In the described VS-MRAC, no explicit differentiations are utilized. The term $(u_0)_{\text{eq}}$ is the equivalent control, which is well described in the variable structure (VS) system (sliding mode) literature.[14] One can formally obtain $(u_0)_{\text{eq}}$ by setting $\dot{e}_0'(t) \equiv 0$ in the dynamic system governing the error e_0'. The signal $(u_0)_{\text{eq}}$ can be approximately obtained from u_0 by means of a low-pass filter with high enough cutoff frequency denominated averaging filter.[14] The inclusion of such averaging filters in the stability analysis has been considered in Refs. 11 and 12. Note that one must choose $\kappa_{\text{nom}} \neq 0$. The block diagram of Fig. 1 gives a better overview of the VS-MRAC structure applied to the wing-rock problem ($n^* = 2$).

Now we state the following result. (A proof is given in the Appendix.)

Theorem 1: Consider the wing-rock model (2) and the VS-MRAC law of Table 2. Then for any trajectory evolving in Ω, the closed-loop system has the following properties.

1) The errors e_i' $(i = 0, 1)$ all converge to zero in finite times.

2) The state error e and the roll angle tracking error e_0 converge exponentially to zero.

Alleviation of the phenomenon of chattering has been treated in Ref. 11. To obtain smooth control signals for variable structure control (VSC) systems, one of the main approaches consists of using a continuous approximation of the switching functions. One example is to replace $u = \text{sgn}(e_i')$ by $u = \text{sat}(e_i')$, where $\text{sat}(\eta)$ is defined as $\text{sat}(\eta) = \text{sgn}(\eta)$ if $|(\eta)| > \Delta$ and $\text{sat}(\eta) = (\eta/\Delta)$ if $|(\eta)| \leq \Delta$. This way chattering of control is avoided; however, this may lead to a small terminal error.

To this end, computation of modulation functions f_i in Table 2 is discussed. The lower bounds of these functions can be computed on-line using the expressions in Table 2 and the available signals $\boldsymbol{\xi}_i$ and χ_0. But the bounds $\bar{g}_i$ on uncertain functions depend on $\boldsymbol{x} \in \Omega$. The set Ω is assumed to contain the trajectories of the system beginning from the set of initial conditions of interest. The existence of a bounded set Ω is easily established because the trajectories of the closed-loop system evolve according to the exponentially stable system (A6) after a finite time, e.g., t^*, and because the trajectories of the wing-rock model (2) with $u = u_{\text{nom}} - f_1\,\text{sgn}(e_1')$ cannot be unbounded in the finite interval $[0, t^*]$. Because the choice of the modulation functions satisfying the inequalities in Table 2 is only sufficient for stability in the closed-loop system, a practical way is to choose an overestimated value of bound $\bar{g}_i$ in the computation of the modulation functions by observing simulated responses of an approximate wing-rock model for a set of initial conditions. A drastic simplification in control law synthesis can be obtained by using constant modulation functions. This is referred to as the Relay VS-MRAC.[11] It should be stressed that this simplification is obtained at the expense of larger modulation functions.

IV. Simulation Results

The uncontrolled system (1) for $\alpha = 25$ deg with the initial condition $\phi(0) = 20$ deg and $\dot{\phi}(0) = 1$ deg s^{-1} was simulated. Figure 2 shows the oscillatory response in the phase plane.

Table 3 Minimum and maximum values of κ^* and θ_i^*

	θ_1^*	θ_2^*	θ_3^*	θ_4^*	κ^*
Fast control					
Minimum value	−2.0105	−3.7644	1.7021	0.8333	0.8000
Maximum value	−2.0042	−2.4981	2.5900	1.2500	1.2000
Slow control					
Minimum value	−2.0210	−0.9323	0.4468	0.8333	0.8000
Maximum value	−2.0084	−0.6178	0.6956	1.2500	1.2000

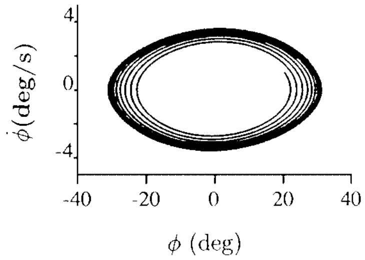


Fig. 2 Wing rock at α = 25 deg.

Now the results of digital simulation for the wing-rock model (2) with the VS-MRAC law given in Table 2 are presented. The initial conditions chosen are $\phi(0) = \phi_m(0) = 20$ deg, $\dot{\phi}(0) = 2$ deg s^{-1}, and $\dot{\phi}_m(0) = 0$. Noting that the controller gains given in Table 2 are only sufficient for stability in the closed-loop system, the values of the modulation functions f_i, τ, and Δ are chosen in several trials by observing simulated responses. For fast control, $L = (s + 1)$, $\lambda = 1$, $\gamma = 1$, and the reference model is

$$M = \frac{1}{(s+1)^2} \tag{19}$$

but for slow control, $L = (s + 0.5)$, $\lambda = 0.5$, $\gamma = 0.5$, and

$$M = \frac{1}{(s+0.5)^2} \tag{20}$$

We note that ML is SPR.

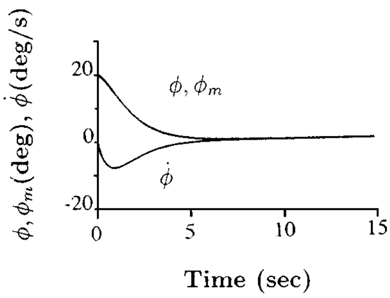
For model matching, the values of $\boldsymbol{\theta}^*$ were computed at angles of attack 21.5, 22.5, and 25 deg using the parameters given in Table 1, assuming that the parameter k_p is uncertain and $k_p \in [0.8, 1.2]$. For these flight conditions, the computed minimum and maximum values of θ_i^* and κ^* for fast and slow control are shown in Table 3.

For the purpose of illustration, we have chosen $\boldsymbol{\theta}_{\text{nom}} = (0, 0, 0, 1)^T$, which is rather an unfavorable choice. In Table 2, various parameters chosen are $\bar{g}_i + \epsilon_i = 1$, $i = 0, 1$, $\bar{\kappa} = 0.2$, and $\kappa_{\text{nom}} = 1$. The parameters $\bar{\theta}_{ij}$ and $\bar{\kappa}$ are chosen so as to satisfy inequalities in Eq. (18) for each value of κ^* and $\boldsymbol{\theta}^*$ in Table 3. Thus, in view of Table 3, for fast control, these are selected as $\bar{\boldsymbol{\theta}}_0 = (2.5, 3.1, 2.1, 0.2)^T$ and $\bar{\boldsymbol{\theta}}_1 = (2.1, 3.8, 2.6, 0.3)^T$, and for slow control, these parameters are $\bar{\boldsymbol{\theta}}_0 = (2.5, 0.8, 0.6, 0.2)^T$ and $\bar{\boldsymbol{\theta}}_1 = (2.1, 1.0, 0.7, 0.3)^T$.

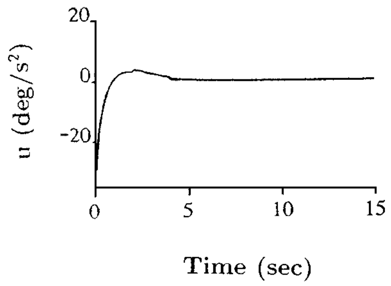
The signal $(u_0)_{\text{eq}}$ is obtained by filtering u_0 by a first-order filter $[1/(\tau s + 1)]$ with $\tau = 0.05$. The sgn function is replaced by the sat function with $\Delta = 0.02$. For simulation it is assumed that the angle of attack and k_p are time varying such that $\alpha = 22.5$ deg and $k_p = 1$ for $t \in [0, 2]$ s, $\alpha = 21.5$ deg and $k_p = 0.8$ for $t \in (2, 4]$ s, $\alpha = 25$ deg and $k_p = 1.2$ for $t \in (4, 6]$ s, and $\alpha = 22.5$ deg and $k_p = 1$ for $t > 6$ s. Because the aerodynamic coefficients are functions of the angle of attack, the parameters of the wing-rock model (2) are assumed to vary in a piecewise manner according to Table 1.

A. Wing-Rock Control: Varying f_i, d = 0

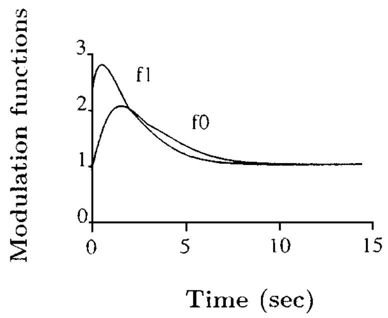
Figures 3a–3c show the selected responses using the reference model (19) for fast control. Time-varying modulation functions are generated on-line using the expressions in Table 2. In spite of the piecewise variation of the parameters of the model, smooth regulation of roll angle is observed in Fig. 3a. The response time is of the order of 6 s. Time-varying modulation functions are shown in Fig. 3c. The roll-angle trajectory follows the reference trajectory, with only a small tracking error in the transient period. The



a) ϕ, ϕ_m, and $\dot{\phi}$



b) Control input



c) Modulation functions, f_i (i = 0, 1)

Fig. 3 Wing-rock control: varying f_i, d = 0.

maximum values of the tracking error e_{0m} and of the control input u_m are 0.36 deg and 29.3 deg s^{-2} (Fig. 3b), respectively.

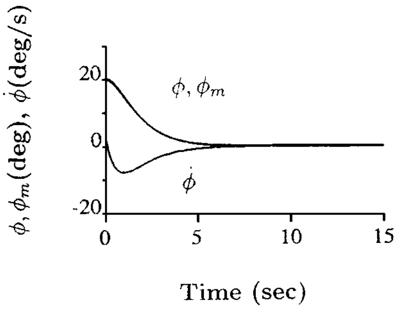
B. Wing-Rock Control: Varying f_i, $d \neq 0$

To examine the effect of disturbance input, a signal $d_w(t)$ was generated by passing a white noise through a filter with gain 2, a zero at $s = -0.2278$, and two poles at $s = -0.3945$; the disturbance input d was assumed to be $d = d_w + 5.72$ (deg/s^2). Figures 4a–4d show selected responses. Variations in the control input are required to cancel the effect of the disturbance input. The responses (Figs. 4a and 4d) remain close to those shown in Fig. 3. Compared with the disturbance-free case A, for the chosen d (Fig. 4c), slightly larger tracking error and control input (Fig. 4b) are observed. The maximum values are $(e_{0m}, u_m) = (0.49$ deg, 32.5 deg s$^{-2})$.

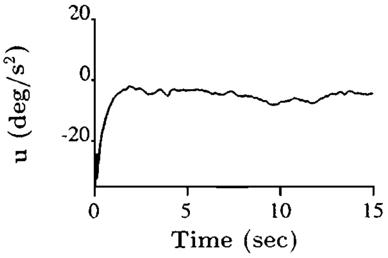
C. Wing-Rock Control: Constant f_i, d = 0 or $d \neq 0$

For simplicity in implementation of the control law, constant modulation functions $f_0 = 2$ and $f_1 = 2.5$, instead of the time-varying functions of cases A and B, are used. The remaining parameters are as in case A. Although one could have chosen values of f_i, $i = 0, 1$, larger than their peak values observed in Figs. 3c and 4d, because the conditions for stability in Table 2 are only sufficient, smaller values of modulation functions were chosen intentionally to avoid high gain feedback. Smooth convergence of the roll-angle trajectory is observed (Fig. 5). The maximum values are $(e_{0m}, u_m) = (0.33$ deg, 31.2 deg s$^{-2})$. For the chosen constant modulation functions, a small increase in input magnitude is required for control; however, compared with case A, the tracking error is smaller in this case.

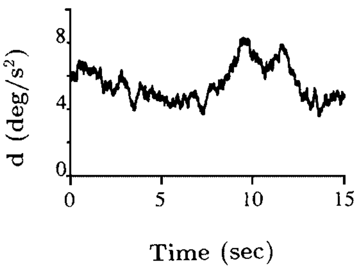
Simulation with constant modulation functions and nonzero disturbance input was also done. The responses remained close to those of case B. The maximum values are $(e_{0m}, u_m) = (0.47$ deg, 33.5 deg s$^{-2})$. (These results are not shown here.)



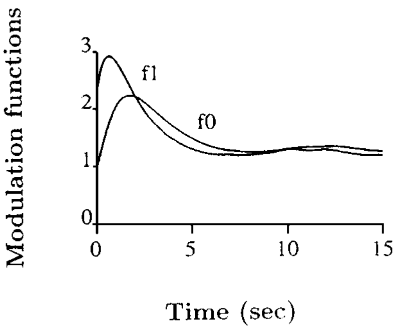
a) ϕ, ϕ_m, and $\dot{\phi}$



b) Control input



c) Disturbance input d



d) Modulation functions, f_i (i = 0, 1)

Fig. 4 Wing-rock control: varying f_i, $d \neq 0$.

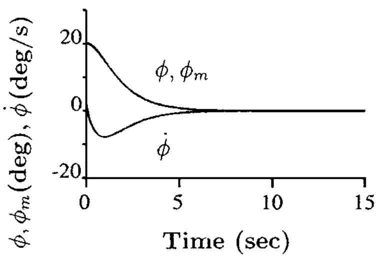
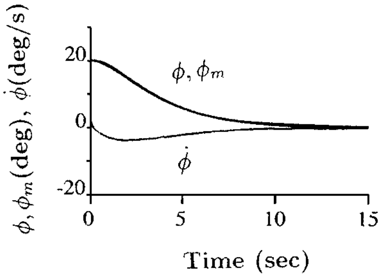
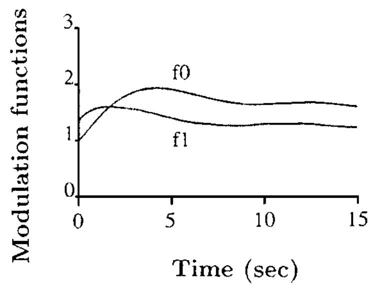


Fig. 5 Wing-rock control: constant f_i, d = 0; ϕ, ϕ_m, and $\dot{\phi}$.



a) ϕ, ϕ_m, and $\dot{\phi}$



b) Modulation functions, f_i (i = 0, 1)

Fig. 6 Wing-rock control: varying f_i, $d \neq 0$, slow command.

D. Wing-Rock Control: Varying $f_i, d \neq 0, d = 0$, Slow y_m

To reduce the control magnitude, a slow command trajectory was generated using the reference model (20). The responses for the model with $d \neq 0$ are shown in Figs. 6a and 6b. The wing-rock motion is suppressed in about 12 s (Fig. 6a) but with considerably smaller control magnitude compared to case B, as predicted. The maximum values are $(e_{0m}, u_m) =$ (0.56 deg, 23.5 deg s^{-2}).

Simulation was also done using the design parameters that were used for Fig. 6, but the disturbance input was assumed to be zero. Smooth regulation of the wing-rock motion was accomplished. The maximum values for the chosen control parameters are $(e_{0m}, u_m) =$ (0.24 deg, 21.4 deg s^{-2}). Again, compared with case A, smaller input is required for control. (These results are not shown here.)

E. Wing-Rock Control: Constant $f_i, d = 0$ or $d \neq 0$, Slow y_m

Simulation was done using the model with and without disturbance input and the parameters of case D, but for simplicity in controller synthesis, the modulation functions were set to constant functions as $f_i = 1.5$, $i = 0, 1$. The responses are somewhat similar to those of case D; therefore, these plots are not shown here. The maximum values for the model without disturbance input are $(e_{0m}, u_m) =$ (0.22 deg, 23.3 deg s^{-2}), and with the disturbance input they are $(e_{0m}, u_m) =$ (0.56 deg, 25.2 deg s^{-2}). Compared with cases A–C for the fast control, smaller control magnitudes are required for the slow control.

V. Conclusions

Based on the variable structure model reference adaptive control theory, a new control law for the control of wing-rock motion was presented. In the wing-rock model, unstructured nonlinearity and disturbance input were present, and the aerodynamic parameters were assumed to be unknown. A variable structure model reference adaptive control system was synthesized using only measurement on the roll angle. In the closed-loop system, roll angle tracked the reference trajectory, and smooth regulation of wing rock was accomplished. Simulation results were presented that showed good transient characteristics and disturbance rejection capability of the designed controller.

Appendix: Proof of Stability

The following lemma from Ref. 12 is used to prove Theorem 1.

Lemma 1: Consider the input–output representation of a single input (U)/single output (q) system given by

$$q(t) = H(s)[-U(t) + w(t) + \pi(t)] \tag{A1}$$

where $H(s)$ is an SPR transfer function, $w(t)$ is a bounded function, and $\pi(t)$ is an exponentially decaying function. If the control input U is chosen as a discontinuous function $U(t) = f(t)\,\mathrm{sgn}(q)$, where f satisfies $f(t) \geq |w(t)| + \epsilon$, $\forall t$, and ϵ is a positive constant, then the state {0} of any state-space realization of Eq. (A1) is globally exponentially stable, and the output $q(t)$ becomes identically zero after some finite time $t_s \geq 0$.

Proof of Theorem 1: This result for wing-rock control is established by following steps similar to those outlined in Ref. 12. For the wing-rock problem, the errors e_0' and e_1' can be written, using Table 2, as

$$e_0' = k_{\text{nom}} ML\Big[-u_0 + (\rho - 1)\big(\chi_0 - L^{-1}u_{\text{nom}}\big) - \rho L^{-1}\big(u^* - u_{\text{nom}}\big) + (k_{\text{nom}}ML)^{-1}\bar{M}g(\boldsymbol{x}, t)\Big] \tag{A2}$$

$$e_1' = \rho L^{-1}\Big[-u_1 - \big(u^* - u_{\text{nom}}\big) + \theta_4^* M^{-1}\bar{M}g(\boldsymbol{x}, t)\Big]$$

Note that the expression for e_1' in Eq. (A2) is obtained by using the definition of e_1' from Table 2 and $(u_0)_{\text{eq}}$ (ignoring exponentially decaying signals) obtained by imposing the sliding-mode condition $e_0'(t) \equiv 0$ in the first equation (A2).

The main steps of the proof are as follows. 1) Because $M(s)L(s)$ is SPR ($k_{\text{nom}} > 0$), in view of Lemma 1 and by the choice of signal u_0 in Table 2, e_0' tends to zero in a finite time. 2) Defining $e_0' = 0$ as a sliding surface,[14] we formally obtain $(u_0)_{\text{eq}}$ by setting $\dot{e}_0'(t) \equiv 0$ in the dynamic system governing the error e_0'. 3) Because $L^{-1}(s)$ is SPR ($\rho > 0$), in view of Lemma 1 and by the choice of u_1 in Table 2, e_1' tends to zero in a finite time. 4) Defining $e_1' = 0$ as a sliding surface, we formally obtain $(u_1)_{\text{eq}}$ by setting $\dot{e}_1'(t) \equiv 0$ in the dynamic system governing the error e_1'. 5) Finally, convergence of $\boldsymbol{e}$ and e_0, which is described in the sequel, is shown.

Consider system (15), which can be rewritten as

$$e_0 = \kappa^* M(s)\Big[\tilde{u} + \theta_4^* M(s)^{-1}\bar{M}(s)g(\boldsymbol{x}, t)\Big] \tag{A3}$$

In view of Eqs. (14) and (15), a state-space realization of Eq. (A3) is

$$\dot{\boldsymbol{e}} = A_c\boldsymbol{e} + \kappa^*\boldsymbol{b}_c\Big[\tilde{u} + \theta_4^* M(s)^{-1}\bar{M}(s)g(\boldsymbol{x}, t)\Big], \qquad e_0 = \boldsymbol{h}_c^T\boldsymbol{e} \tag{A4}$$

Noting that $\tilde{u} = u - u^*$ and $u = -u_1 + u_{\text{nom}}$, Eq. (A2) gives

$$\Big[\tilde{u} + \theta_4^* M(s)^{-1}\bar{M}(s)g(\boldsymbol{x}, t)\Big] = [(s + \delta)/\rho](e_1') \tag{A5}$$

Then defining $\bar{\boldsymbol{e}} = \boldsymbol{e} - \kappa_{\text{nom}}\boldsymbol{b}_c e_1'$ and using Eq. (A4), we have

$$\dot{\bar{\boldsymbol{e}}} = A_c\bar{\boldsymbol{e}} + \kappa_{\text{nom}}(A_c + \delta I)\boldsymbol{b}_c e_1' \tag{A6}$$

Because A_c is exponentially stable and $e_1'(t)$ tends to zero in a finite time, we conclude that $\|\bar{\boldsymbol{e}}(t)\|$ and, consequently, $\|\boldsymbol{e}(t)\|$ tend exponentially to zero. From $e_0(t) = \boldsymbol{h}_c^T\boldsymbol{e}(t)$, we also have that $e_0(t)$ tends exponentially to zero. This completes the verification of properties 1 and 2.

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